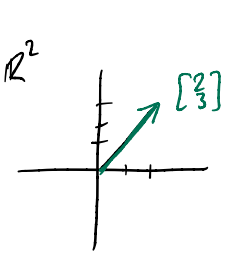
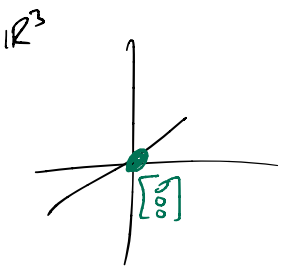
Vectors

An **n-dimensional vector** is a vertical list of n numbers. The set of all n-dimensional vectors is denoted by $\mathbb{R}^n$

Ex: $v = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 5 \end{bmatrix} \in \mathbb{R}^4$, $w = \begin{bmatrix} 1 \\ -1 \\ 2.5 \end{bmatrix} \in \mathbb{R}^3$ (can also write as $v = (1, 0, -1, 5)$)

Geometrically





Basic Operations

Addition: Let $v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ and $w = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$.

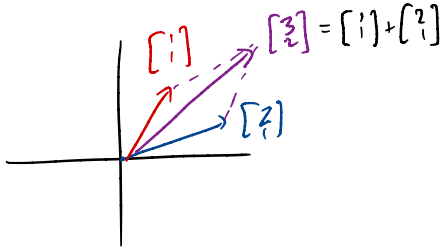
Then $v + w = \begin{bmatrix} v_1 + w_1 \\ \vdots \\ v_n + w_n \end{bmatrix}$

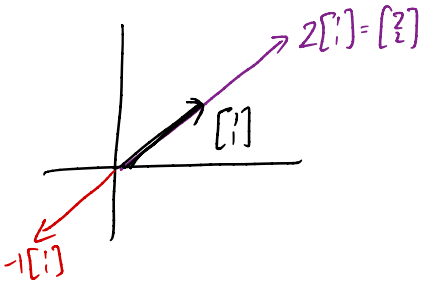
Scalar Multiplication: Let c be a real number (scalar) and $v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$.

Then $cv = \begin{bmatrix} cv_1 \\ \vdots \\ cv_n \end{bmatrix}$

Note: Define subtraction by $v - w = v + (-1 \cdot w)$

Geometrically:





Properties:

- $v+w = w+v \quad \forall\ v,w \in \mathbb{R}^n$ (commutative)

  proof: Since addition occurs entrywise and addition of scalars is commutative:

  Let $v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ and $w = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$.

  Then $v+w = \begin{bmatrix} v_1+w_1 \\ \vdots \\ v_n+w_n \end{bmatrix} = \begin{bmatrix} w_1+v_1 \\ \vdots \\ w_n+v_n \end{bmatrix} = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} = w+v$

- $v+(w+u) = (v+w)+u \quad \forall\ v,w,u \in \mathbb{R}^n$ (associative)

  proof: Since addition occurs entrywise and addition of scalars is associative

- $a(v+w) = av+aw \quad \forall\ v,w \in \mathbb{R}^n,\ a \in \mathbb{R}$ (distributive)

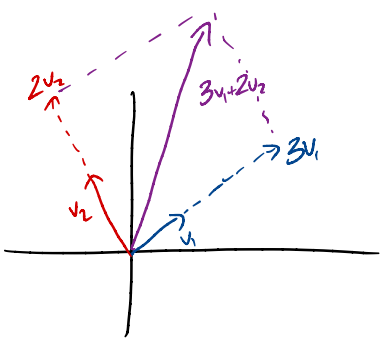
  proof: Since addition/scalar multiplication occurs entrywise and distribution holds for scalars

Let $v_1, v_2, \dots, v_k \in \mathbb{R}^n$ and $c_1, \dots, c_k \in \mathbb{R}$

then $c_1v_1 + c_2v_2 + \cdots + c_kv_k$ is a linear combination of $v_1, \dots, v_k$ with weights $c_1, \dots, c_k$

Ex: Let $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

Then $3v_1 + 2v_2 = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$



this concept underlies much of linear algebra. We will develop an algorithm to systematically solve questions like:

Express $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 3/2 \\ 0 \\ -2 \end{bmatrix}$ (if possible)

Dot Product: Takes $v, w \in \mathbb{R}^n$ and gives $v \cdot w \in \mathbb{R}$.

If $v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$, $w = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} \in \mathbb{R}^n$, then $v \cdot w = v_1w_1 + \cdots + v_nw_n$

Ex: $\begin{bmatrix} -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 5 \end{bmatrix} = (-1)(1) + 2(5) = 9$

We will see the geometry of dot products later in the semester.